THEORETICAL AND EXPERIMENTAL STUDIES OF THE NATURE AND CHARACTERISTICS OF SPACE-RELATED PLASMA RESONANCE PHENOMENA

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SUMMARY

The subjects of this research grant are theoretical and experimental studies of the nature and characteristics of space-related plasma resonance phenomena, and are proceeding under the direction of Dr. F. W. Crawford. This is the first semi-annual progress report on the work and covers the period from May 1 to October 31, 1965.

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I. INTRODUCTION

In recent years, the rapid advance of space technology has made plasma regions such as the ionosphere accessible to direct study, and has opened the way to wide-ranging series of experiments with space-probing vehicles. In some cases, the data have confirmed previous knowledge obtained by ground-based apparatus, such as transmitters, receivers, spectrometers, etc., and in others have produced evidence of unexpected phenomena. The work carried out so far under this contract stems from observations in the latter class, and is related to results obtained by the Canadian top-side sounder satellite, "Alouette."[1-7]

The primary objective of this launching in September 1962 was to determine part of the electron density profile in the ionosphere by bouncing radio signals off it from above. Amongst the equipment carried was a pulsed transmitter of variable frequency. Apart from the expected reflection of its signals from the ionosphere, it was found that at certain discrete frequencies a pronounced ringing effect occurred, lasting for many rf periods after the end of the transmitter pulse. It was found that the frequencies involved corresponded extremely closely to harmonics of the local electron cyclotron frequency.

The origin of the resonances was at first puzzling, since no cyclotron harmonic phenomena are predicted by the linear, cold-plasma theory normally applied to ionospheric plasma propagation phenomena, and a search was made to see whether any similar phenomenon had been observed and explained in laboratory plasma experimentation. As it happened, data on ion and electron cyclotron harmonic noise emission, and absorption from magnetoplasmas had been accumulating since 1959, and in 1963 it was suggested independently by Canobbio and Croci,[8] and Tanaka et al.,[9] that these might be explicable in terms of the warm magnetoplasma theory developed in the 1950's and presented in final form by Bernstein in 1958.[10] This predicts, for both electrostatic and electromagnetic waves, propagation "windows" located at the cyclotron harmonic frequencies. The existence of these was verified at Stanford in early 1964 for electrostatic waves propagating perpendicular to the magnetic field.[11] Careful examination of the work carried out up to about that time showed that almost all of the results could be readily explained by warm magnetoplasma theory.[12] Since then, a great many papers have been published in the field. Some of these will be referred to in this report. About thirty others have been summarized in a recent report on European travel partially supported by this NASA Research Grant.[13]

The broad objectives of our current research program are to reproduce in the laboratory the main features of the warm magnetoplasma dispersion relations, including the "Alouette" resonances, and to determine the limits to which these can be made to agree with theory. If the accuracy can be made high, a whole new range of ionospheric diagnostic techniques will be opened up for such quantities as magnetic field, charged particle density and temperature, and collision frequency. The detailed objectives will now be discussed in relation to the projects studied during the reporting period.

II. CURRENT RESEARCH PROGRAM

A. THEORETICAL STUDIES OF CYCLOTRON HARMONIC WAVE PROPAGATION IN PLASMAS

Project No. 1308 — F. W. Crawford, R. Bruce, J. A. Tataronis

Propagation in a warm magnetoplasma is described by Maxwell's equations in the form

$$\nabla \times \underset{\sim}{E} = -j\omega\mu_0 \underset{\sim}{H} \quad ,$$
$$\nabla \times \underset{\sim}{H} = j\omega \overset{\leftrightarrow}{\epsilon}_p \cdot \underset{\sim}{E} \quad , \tag{1}$$

where the time variation is as $\exp j\omega t$, and the other symbols have their usual meaning. The space variation will be taken as $\exp(-j\, \underset{\sim}{k} \cdot \underset{\sim}{r})$. The form of the plasma permittivity tensor, $\overset{\leftrightarrow}{\epsilon}_p$ is given, for example, by Stix,[14] but here we shall be interested only in those terms required to describe the propagation of longitudinal modes $(\underset{\sim}{E} \| \underset{\sim}{k})$. For an isotropic plasma of temperature T , solution of Eq. (1) yields the dispersion relation[14]

$$k_{\|}^2 + k_{\perp}^2 \left[1 + \frac{\omega_p^2}{\omega_c^2} \sum_{-\infty}^{+\infty} \frac{\exp(-\lambda)\, I_n(\lambda)}{\lambda} A_n \right] = 0 \quad , \tag{2}$$

where $k_{\|}$ and $k_{\perp}$ are the components of $\underset{\sim}{k}$ parallel and perpendicular to the static magnetic field; ω_p and ω_c are the electron plasma and cyclotron frequencies; λ is a parameter replacing $(k_{\perp}R)^2$ in which R is the Larmor radius of a particle with thermal energy $[R = (\kappa T/m)^{1/2}/\omega_c]$, and A_n is defined by,

$$A_n = 1 + \frac{\omega}{k_{\|}}\left(\frac{\pi m}{2\kappa T}\right)^{1/2} \exp(-\alpha_n^2)\left\{ j\frac{k_{\|}}{|k_{\|}|} - \frac{2}{\pi^{1/2}}\int_0^{\alpha_n} \exp t^2 dt \right\} , \qquad (3)$$

and,

$$\alpha_n = \frac{\omega + n\omega_c}{k_{\|}}\left(\frac{m}{2\kappa T}\right)^{1/2} . \qquad (4)$$

Undamped propagation is predicted perpendicular to the magnetic field, according to the dispersion relation,

$$1 = \frac{\omega_p^2}{\omega_c^2}\sum_{n=1}^{\infty} \frac{\exp(-\lambda)\ I_n(\lambda)}{\frac{\lambda}{2}\left[\left(\frac{\omega}{n\omega_c}\right)^2 - 1\right]} . \qquad (5)$$

For oblique propagation $(k_{\|} \neq 0 \ , \ k_{\perp} \neq 0)$ cyclotron and Landau damping will occur.

At the start of our investigations of cyclotron harmonic waves at Stanford, in early 1964, under NSF sponsorship, the only published numerical plots of the dispersion relations expressed by Eqs. (2) and (5) were those of Bernstein.[10] These were for Eq. (5) and were effectively of (ω/ω_c) against $(k_{\perp}L_D)$ for fixed $(k_{\perp}R)$, where L_D, is the electronic Debye length $[(\kappa T/m)^{1/2}/\omega_p]$. In ionospheric propagation experiments, where ω_p and ω_c will remain fixed, it is more logical to obtain plots of (ω/ω_c) against $(k_{\perp}R)$ for fixed (ω_p/ω_c). These were obtained using an integral form of Eq. (5) to facilitate computation and are reproduced in Fig. 1.[15] For laboratory experiments it is more likely that ω_p and ω will be fixed as ω_c, i.e., the magnetic field, and $k_{\perp}$ are varied, or that ω_c and ω_p^2, i.e., the electron density,

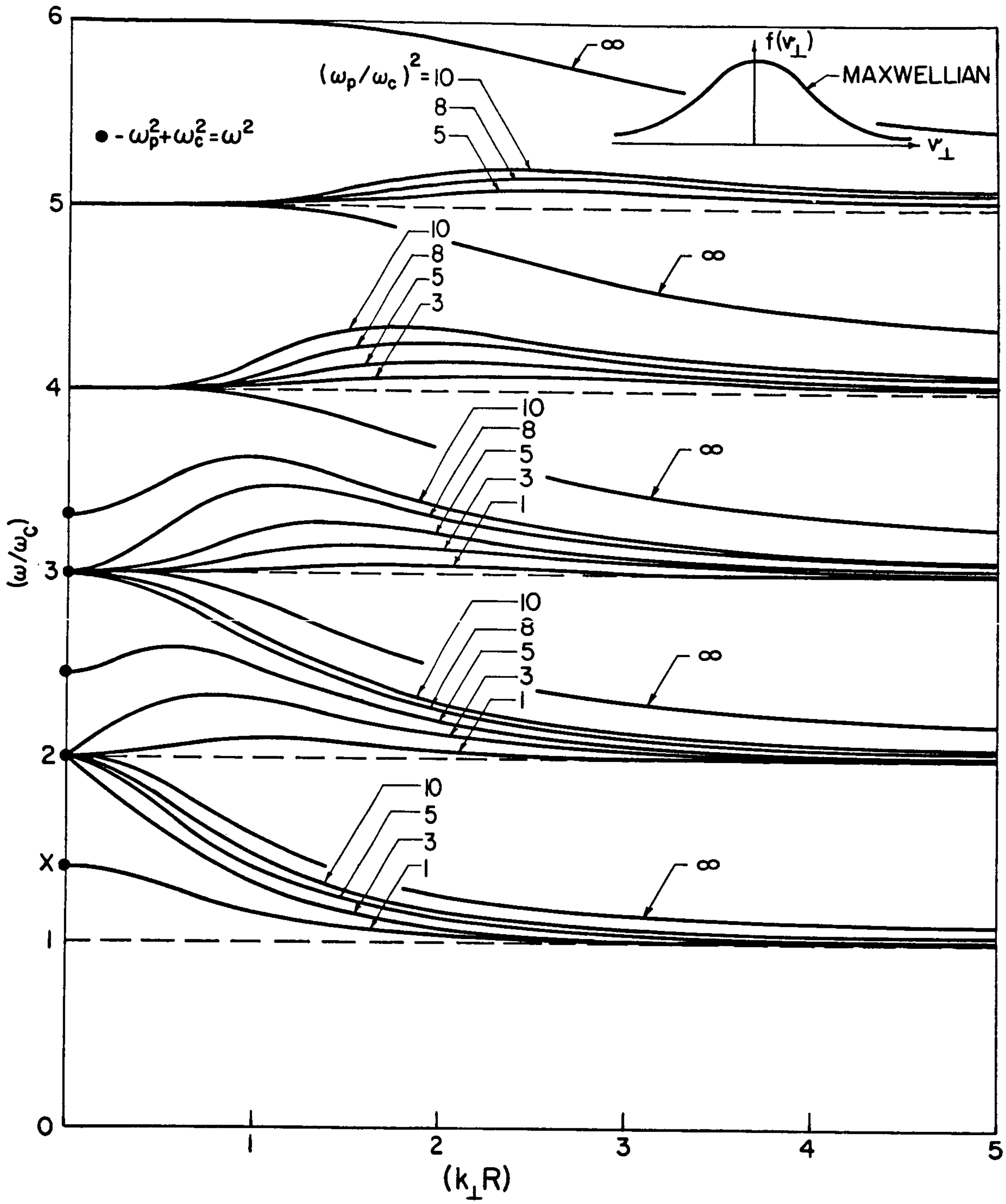


FIG. 1 Dispersion curves for perpendicularly-propagating cyclotron harmonic waves in a Maxwellian plasma - (ω_p^2/ω_c^2) constant

will be varied for fixed ω and $k_\perp$. Plots appropriate to such conditions are reproduced in Figs. 2 and 3.

Under the present contract, three extensions to the above work are envisaged. The first is a study of the effects of collisions. It is evident from Figs. 1 and 2 that the group velocity of the waves is generally small for long and short wavelengths, i.e., small and large $k_\perp$, and near the propagation frequency maxima occurring in the various passbands. Low group velocity implies high collisional attenuation, so that even though perpendicular propagation is free from cyclotron and Landau damping there may be strong collisional damping. Since the generally accepted explanation of the "Alouette" resonances is now that they correspond to points of approximately zero group velocity, it is essential to assess the damping under such conditions. The second addition required to improve our understanding of cyclotron harmonic wave propagation is a numerical study of the dispersion relation of Eq. (2) for oblique propagation to determine the effectiveness of cyclotron and Landau damping. The third extension that it is planned to make to the theory is a study of the antenna excitation of these waves. Some work carried out along this line has already been published.[7] Further development is required to extend this to practical antennas, and measurements, such as of radiation impedance, that can be made with them.

During this reporting period, progress on the first two topics mentioned has been made, and will now be described. The third problem has not yet been tackled. It may be remarked here that the quasistatic dispersion relation will be expected to break down as $|k| \rightarrow 0$. Studies of the dispersion relation under this condition have been described recently.[16,17] In our case, it is felt that the range of k-values available experimentally to provide propagation and ringing effects will be sufficiently great for these effects to be observable. For this reason our analyses are restricted at present to propagation within the quasistatic, or slow-wave approximation.

Effect of Collisions on Perpendicular Propagation

Two distinctly different approaches to the dispersion relation of Eq. (5) may be taken. Either the full tensor treatment leading to Eq. (1)

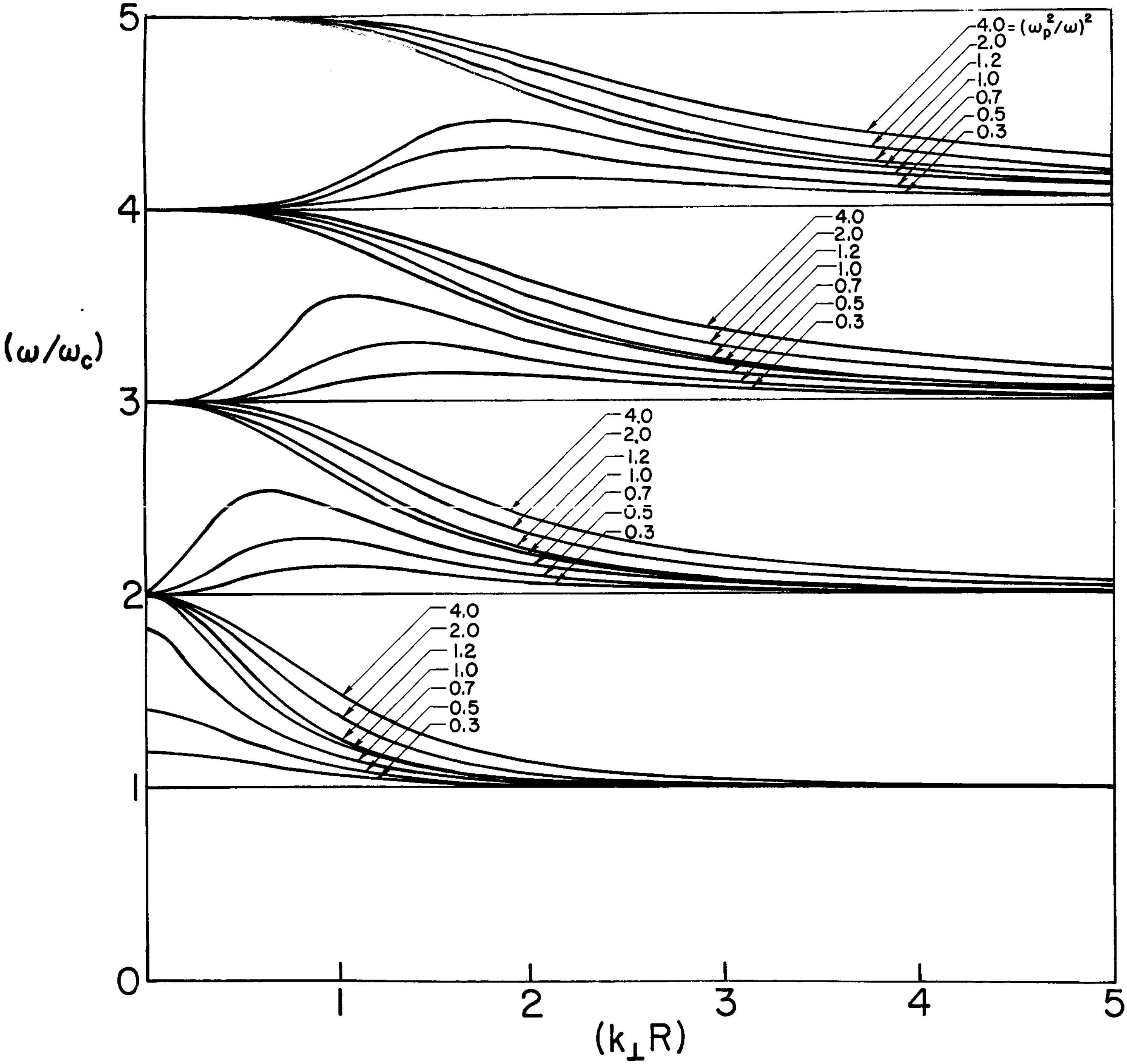


FIG. 2 Dispersion curves for perpendicularly-propagating cyclotron harmonic waves in a Maxwellian plasma - (ω_p^2/ω^2) constant

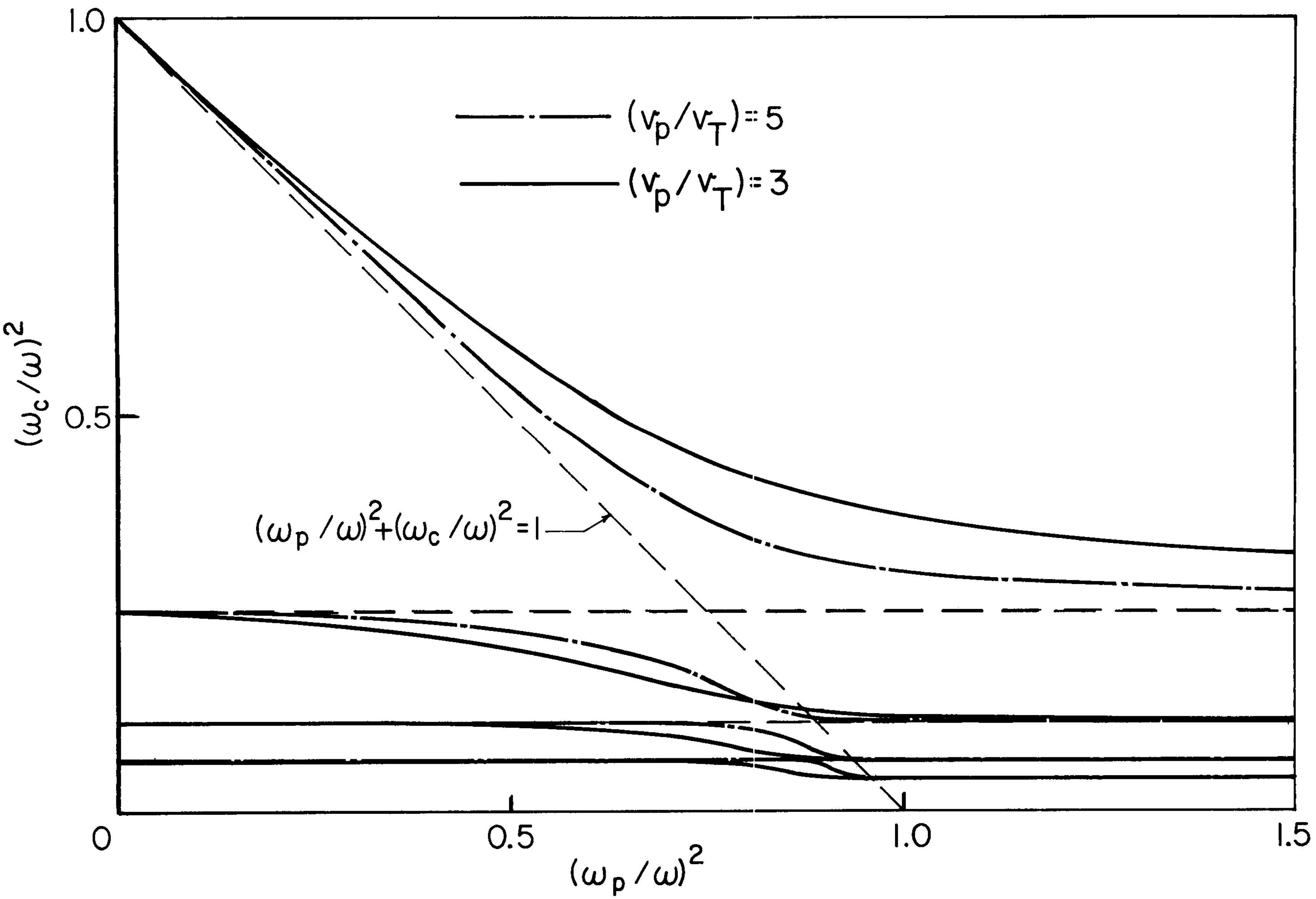


FIG. 3 Dispersion curves for perpendicularly-propagating cyclotron harmonic waves in a Maxwellian plasma $-(v_p/v_T)$ constant, where v_T is the thermal velocity $[=(kT/m)^{1/2}]$ and v_p is the phase velocity $[=\omega/k_\perp)]$

may be carried through, in which case the first moment of the Boltzmann equation is used to obtain the plasma convection current,[10,14] or an electrostatic treatment can be used directly.[15,18] In this case, the rf charge density, i.e., zeroth moment, is calculated and substituted in Poisson's equation. It turns out that the second method requires more care in the choice of form for the collision term in the Boltzmann equation if the correct result is to be obtained. In the first case, the commonly employed form,

$$\left(\frac{df}{dt}\right)_c = \nu[f_0 - f] \quad , \tag{6}$$

is adequate, where ν is the electron-neutral collision frequency, f_0 and f are the unperturbed and perturbed electron velocity distributions. In the second, since we are calculating rf charge density, it is essential to employ a collision term that allows the particles to relax in position space to the local density $(n_0 + n_1)$ rather the unperturbed density, n_0, corresponding to f_0. A suitable approximation for the collision term is,[19]

$$\left(\frac{df}{dt}\right)_c = \nu\left[f_0\left(1 + \frac{n_1}{n_0}\right) - f\right] \quad . \tag{7}$$

The subsequent analysis has been given elsewhere,[20] and leads to the modified dispersion relation,

$$1 = \left(1 - j\frac{\nu}{\omega}\right)\frac{\omega_p^2}{\omega_c^2}\sum_{n=1}^{\infty}\frac{\exp(-\lambda)\, I_n(\lambda)}{\frac{\lambda}{2}\left[\left(\frac{\omega - j\nu}{n\omega_c}\right)^2 - 1\right]} \quad . \tag{8}$$

Solutions to this could now be obtained for ω complex and $k_\perp$ real, or for ω real and $k_\perp$ complex. It is clear that exact computation of the former is relatively uninteresting: for all values of $k_\perp$, there will be a temporal decrement of order (ν/ω). The latter is of considerable interest since it is appropriate to signals propagating away from a source. Computer calculations based on Eq. (8) have been carried out for this case. To begin the discussion of them, it is convenient to start from the solution to Eq. (8) with $\nu = 0$. Typical curves are shown in Fig. 4. The effects of introducing collisions are illustrated in Fig. 5. It will be noted that for the real part of $k_\perp$ (Fig. 5a), collisons have little effect at large values of $k_\perp$. For small $k_\perp$, however, the solutions no longer pass through the points $(\omega/\omega_c) = n$ $(n = 2,3,\ldots..)$. Examination of the imaginary part of $k_\perp$ (Fig. 5b) indicates infinite attenuation as the harmonics are approached, and minima corresponding closely to points where the group velocity is maximum. It is worth noting that, numerically, equal real and imaginary parts of $k_\perp$, or a value of unity for $(k_\perp R)_i$, imply attenuation of over 50 dB/wavelength or/gyroradius, respectively. The implication is that collision damping may be extremely high over wide ranges of $(k_\perp R)_r$ unless (ν/ω_c) is extremely small.

Cyclotron and Landau Damping

Equation 2 states the dispersion relation for oblique propagation. Computations of it are very difficult to program and tedious to carry out, first because the integral formulation feasible for the case of perpendicular propagation is no longer applicable, and second because each term of the infinite series involves evaluation of the complex error function. However, after considerable effort, the dispersion function has been programmed to give accurate solutions. The routines for the complex error function evaluation have been checked against published tables, and work of Derfler and Simonen carried out at Stanford.

Figure 6 shows solutions for two values of (ω_p^2/ω_c^2). In these $\underset{\sim}{k}$ has been assumed to be real, and complex values of ω have been

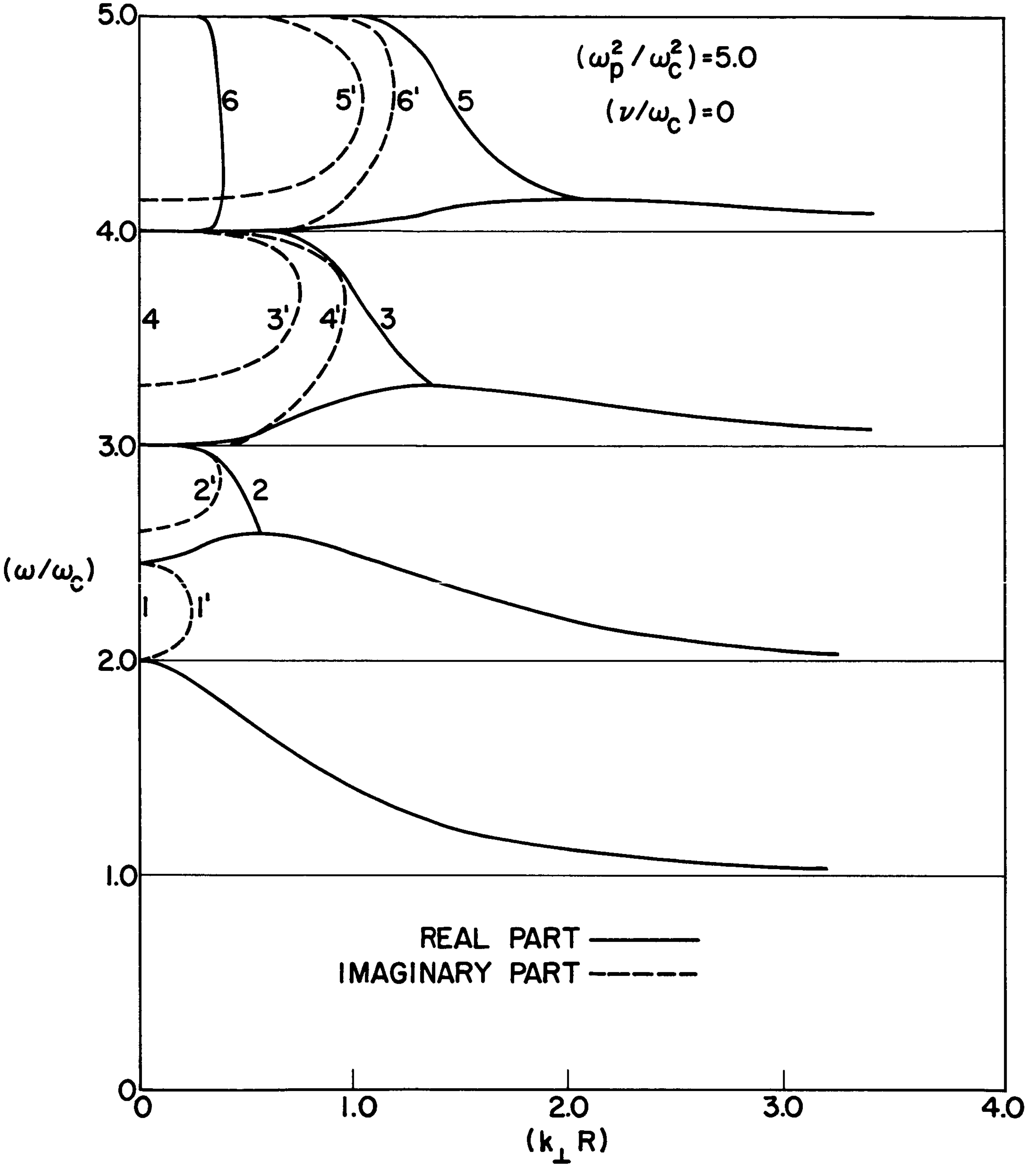


FIG. 4 Dispersion curves for perpendicularly-propagating cyclotron harmonic waves in a Maxwellian plasma - $(\nu/\omega_c) = 0$

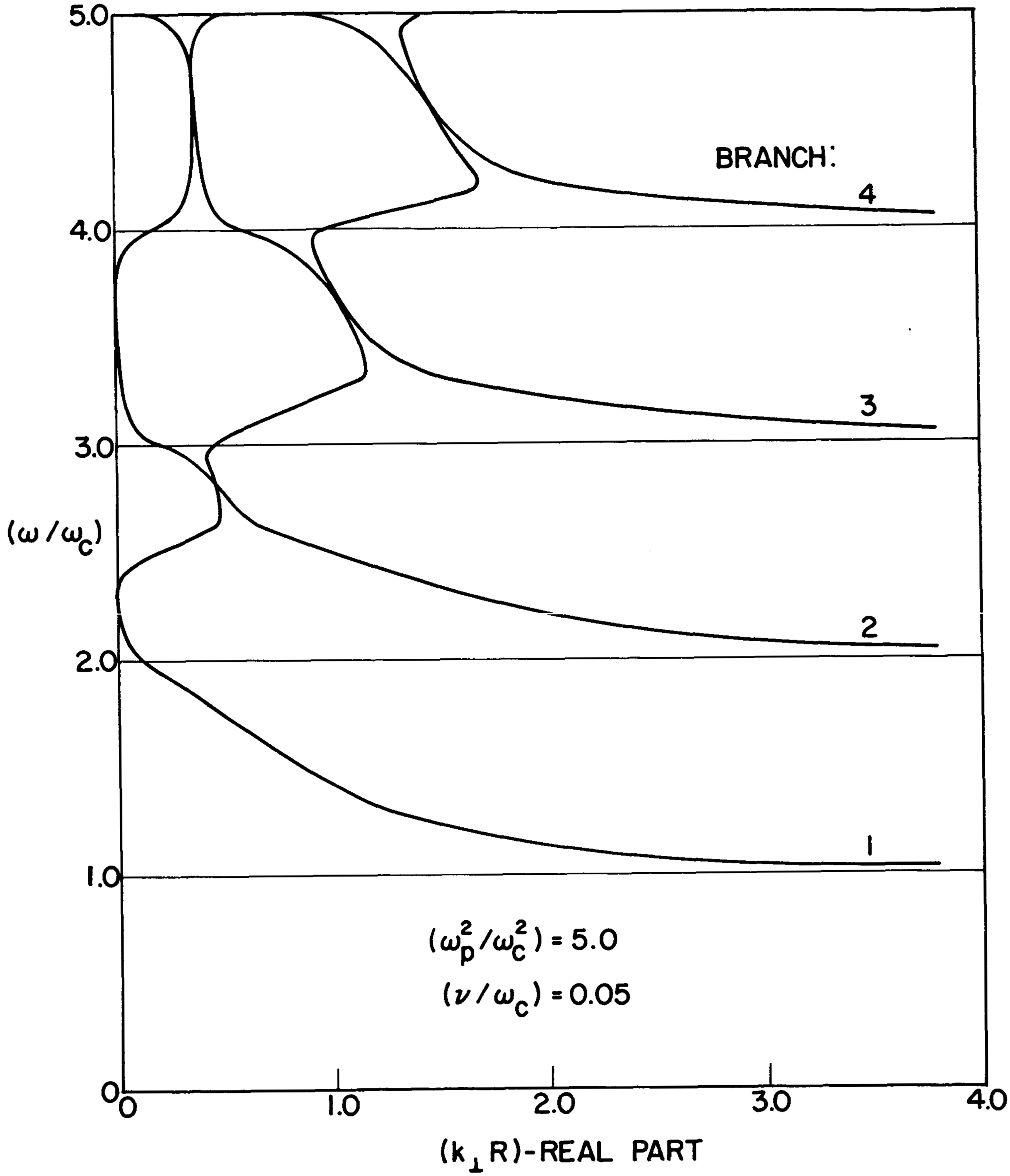


FIG. 5a Dispersion curves for perpendicularly-propagating cyclotron harmonic waves in a Maxwellian plasma - $(\nu/\omega_c) = 0.05$ [Note: the imaginary parts of (ω/ω_c) are displaced vertically and have been plotted relative to the appropriate base-line $(\omega/\omega_c) = n$]

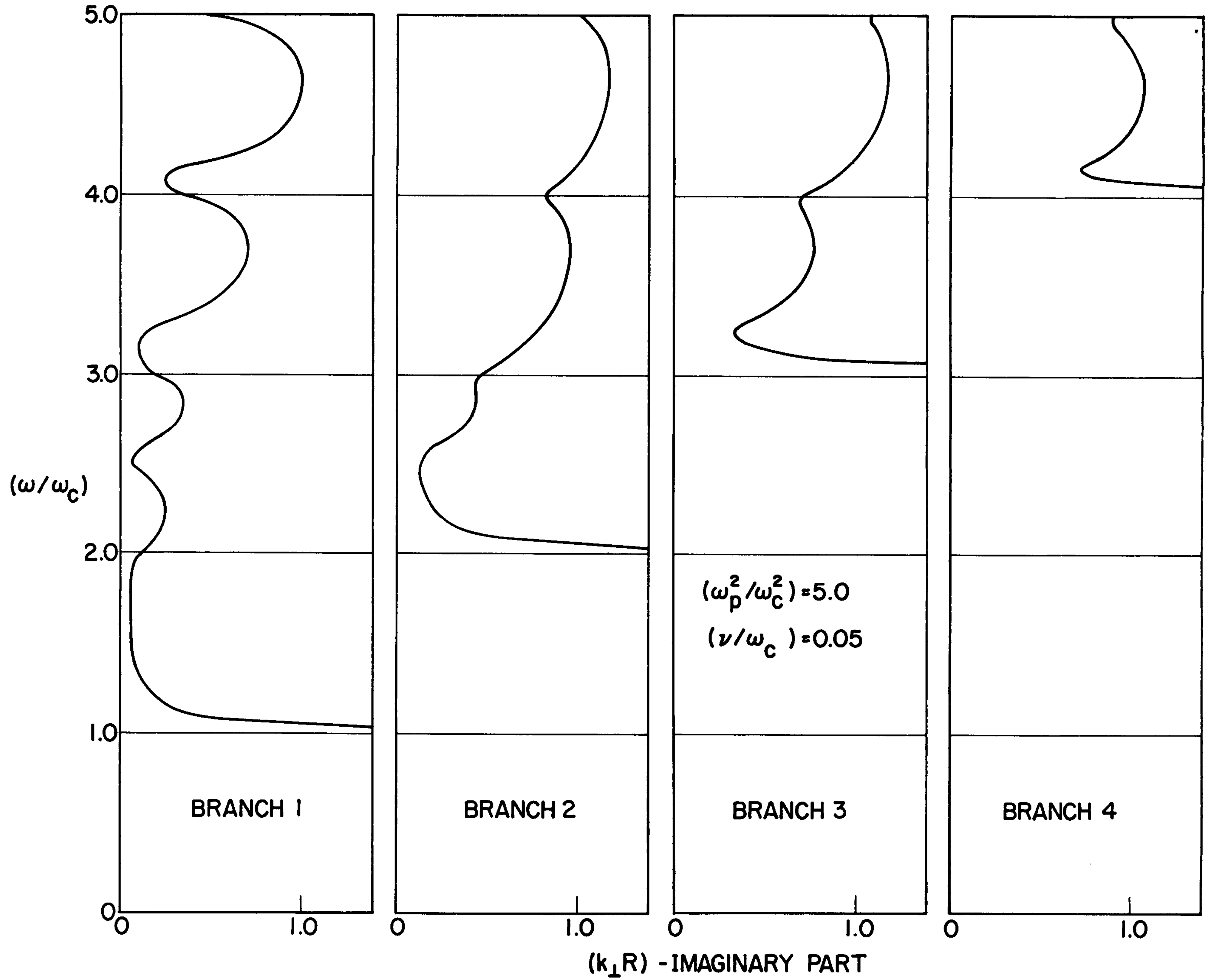

5.0
4.0
3.0
2.0
1.0
0
(ω/ω_c)
0
1.0
BRANCH 1
BRANCH 2
BRANCH 3
BRANCH 4
$(\omega_p^2/\omega_c^2) = 5.0$
$(\nu/\omega_c) = 0.05$
$(k_\perp R)$ - IMAGINARY PART

FIG. 5b

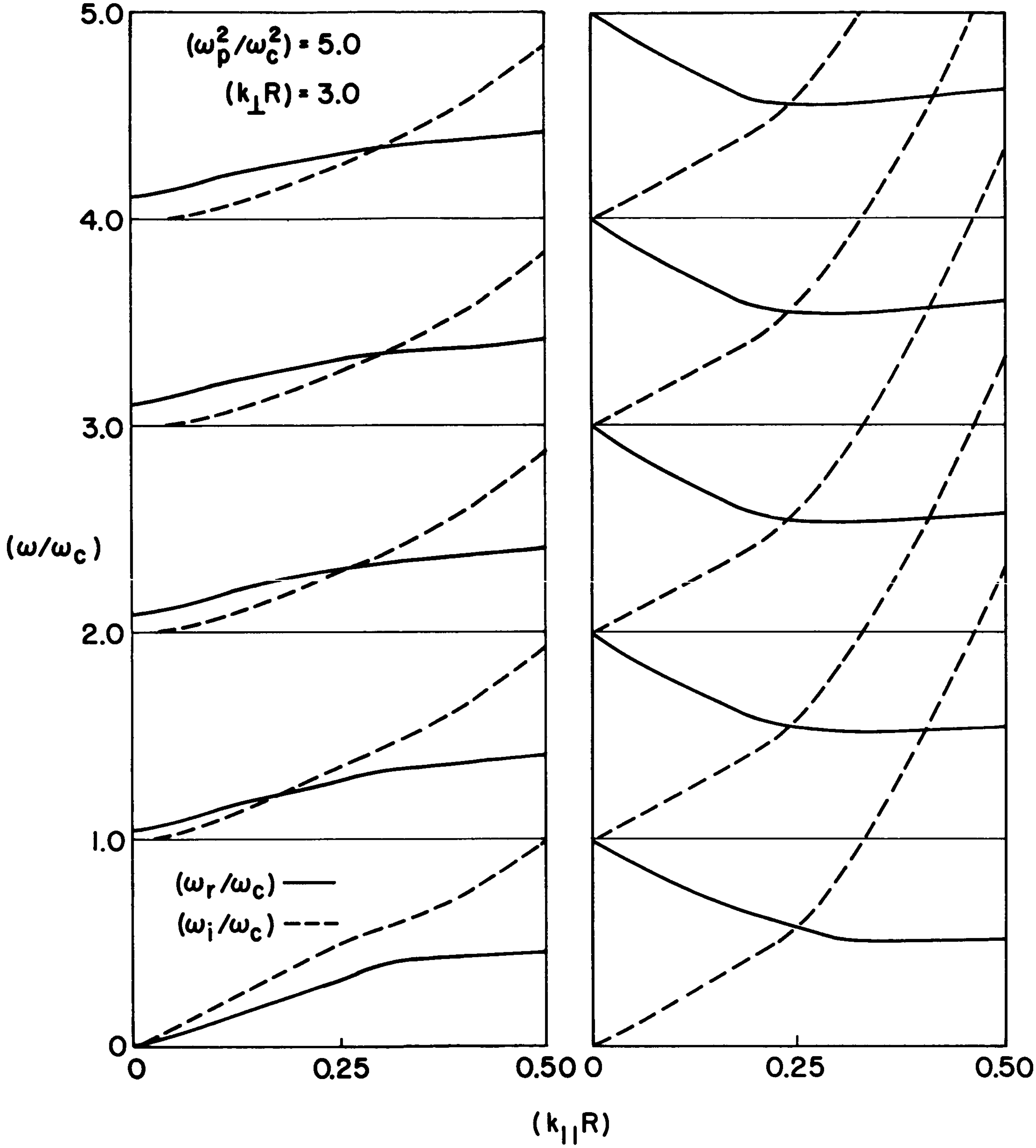
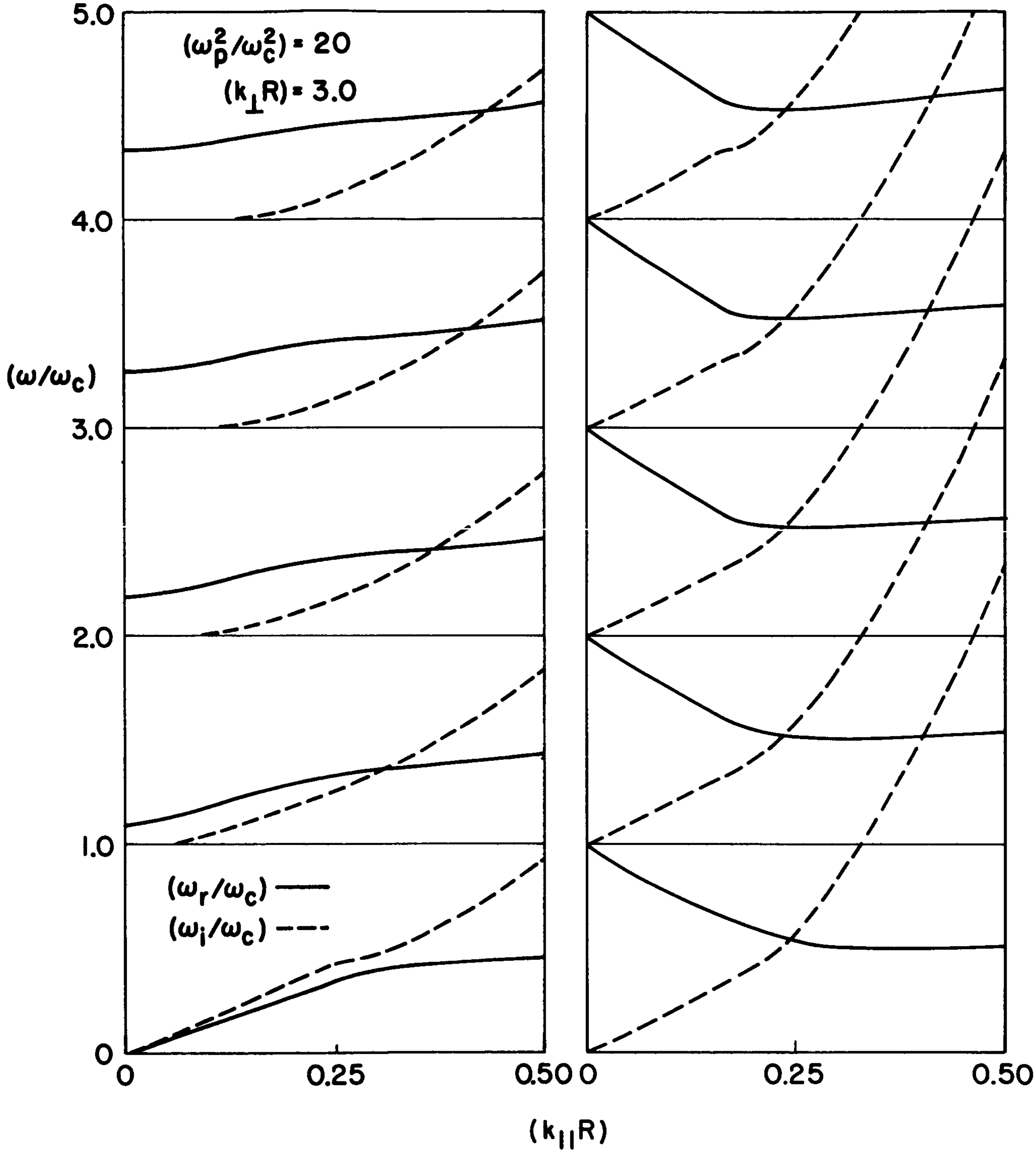


FIG. 6a Dispersion curves for obliquely-propagating cyclotron harmonic waves in a Maxwellian plasma showing cyclotron and Landau damping effects



$(\omega_p^2/\omega_c^2) = 20$
$(k_\perp R) = 3.0$
(ω/ω_c)
5.0
4.0
3.0
2.0
1.0
0
(ω_r/ω_c)
(ω_i/ω_c)
0
0.25
0.50
0
0.25
0.50
$(k_{||}R)$

FIG. 6b

computed. It will be noted that two solutions are obtained in each passband, and that the imaginary parts increase very rapidly with $(k_\parallel R)$, particularly for downgoing solutions. Again it is worth remarking that, numerically, a value of unity for $(\omega/\omega_c)_i$ implies attenuation of over 50 dB/cyclotron period. The implication is that, for propagation more than about 10° off exact perpendicularity, attenuation of this order would be encountered, i.e. wave propagation will normally only be observable in a narrow angular range near $(\pi/2)$ to the magnetic field.

B. EXPERIMENTAL STUDIES OF CYCLOTRON HARMONIC WAVE PROPAGATION IN PLASMAS

Project No. 1309 — F. W. Crawford, R. S. Harp, and H. H. Weiss

The main purpose of this project is to verify the predictions of Eqs. (2), (5) and (8). If the measured and theoretical dispersion relations are sufficiently close, it should be possible to use cyclotron harmonic waves for measurement of plasma and cyclotron frequency, i.e. electron density and magnetic field; electron temperature, and collision frequency. In the ionosphere, where collisions are comparatively rare, it may also be possible to extend the technique to measurements of these parameters relevant to the various ionic constituents.

Our initial experiments are a continuation of those begun in 1964[11] under an NSF grant which has since expired. A schematic of the experimental discharge tube is shown in Fig. 7. Mercury-vapor at a pressure of about 10^{-3} mm Hg is used as the working gas. An rf signal is applied to one antenna, and the signal received on the other is recorded for different values of discharge current and magnetic field. Typical records are as shown in Fig. 8. It has been pointed out by Harp[21,22] that this measurement is effectively a determination of the variation of impedance of the plasma between the two antennas. In principle, these measurements may serve to verify the correctness of the cyclotron harmonic wave dispersion relation since this is involved in the computation of the impedance, which is determined by the following sequence of steps. Charges +Q and -Q are assumed on the two antennas. Since the geometry is known, this effectively determines the electric displacement, D as a function of the space coordinates. Fourier

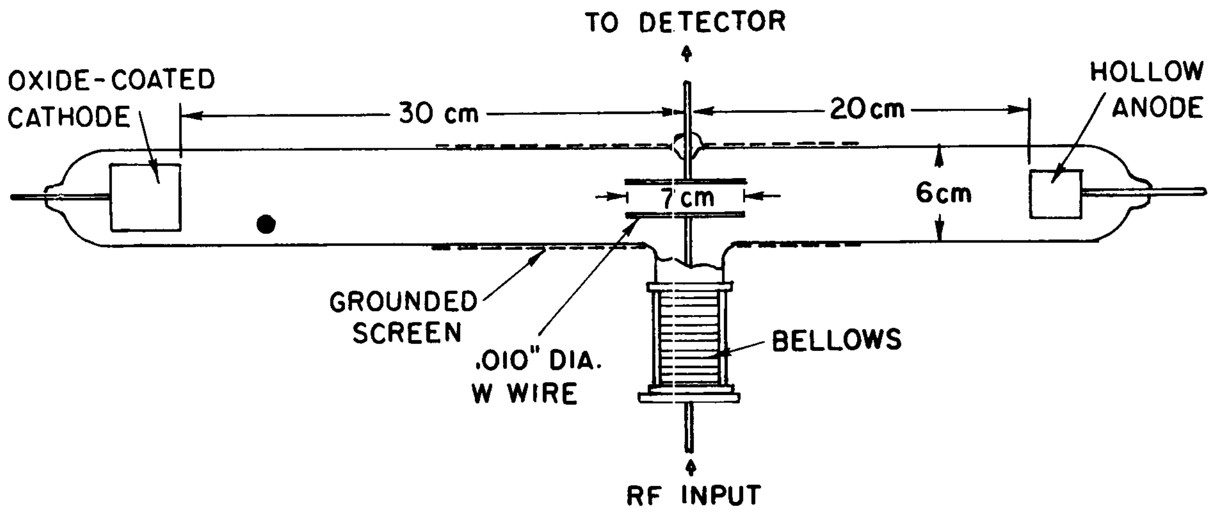


FIG. 7 Experimental mercury-vapor discharge tube for cyclotron harmonic wave propagation studies

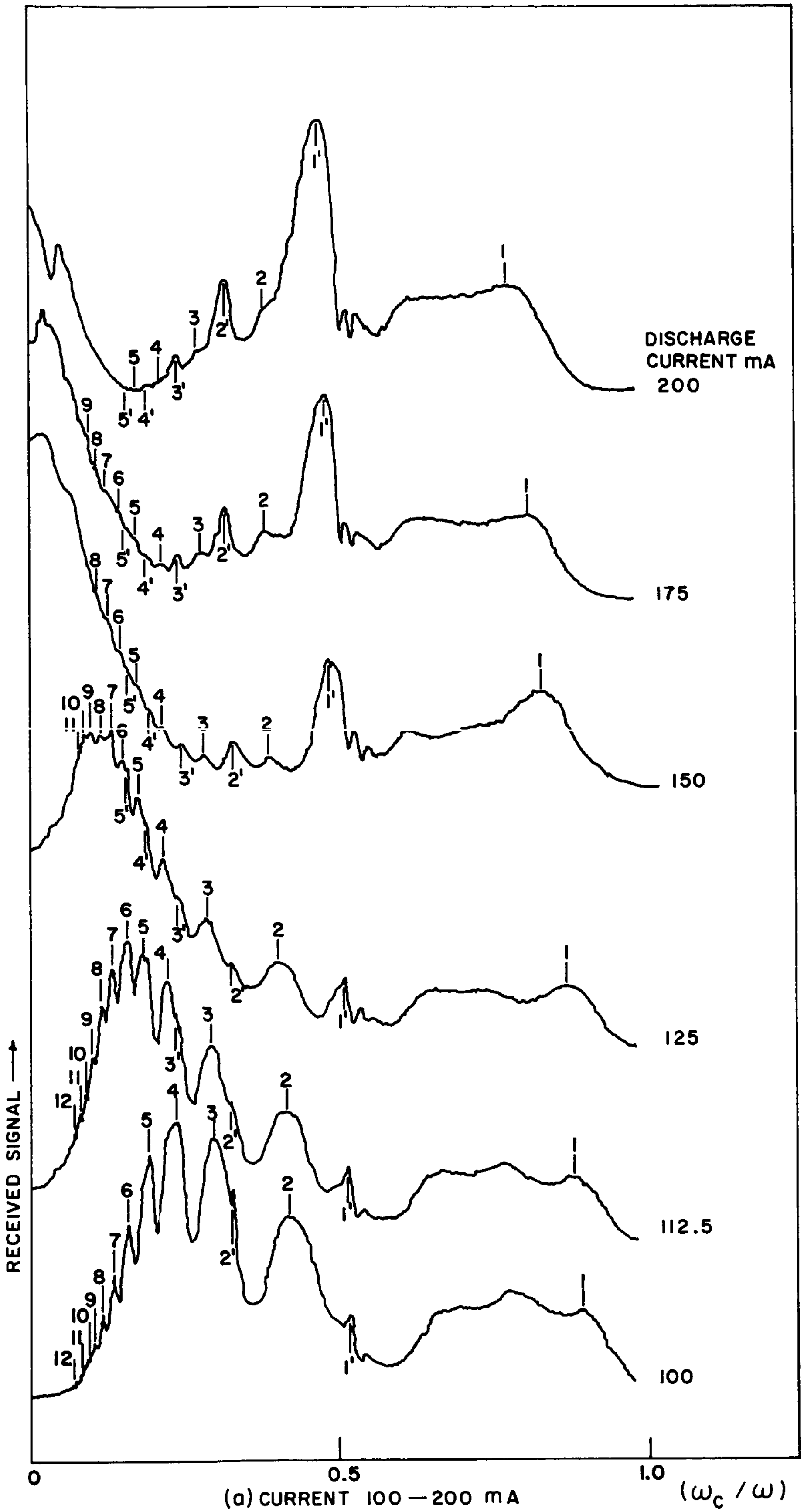


FIG. 8a Transmission curves with varying current. Mercury vapor at $\sim 10^{-3}$ mm Hg. (the curves have been separated vertically for clarity

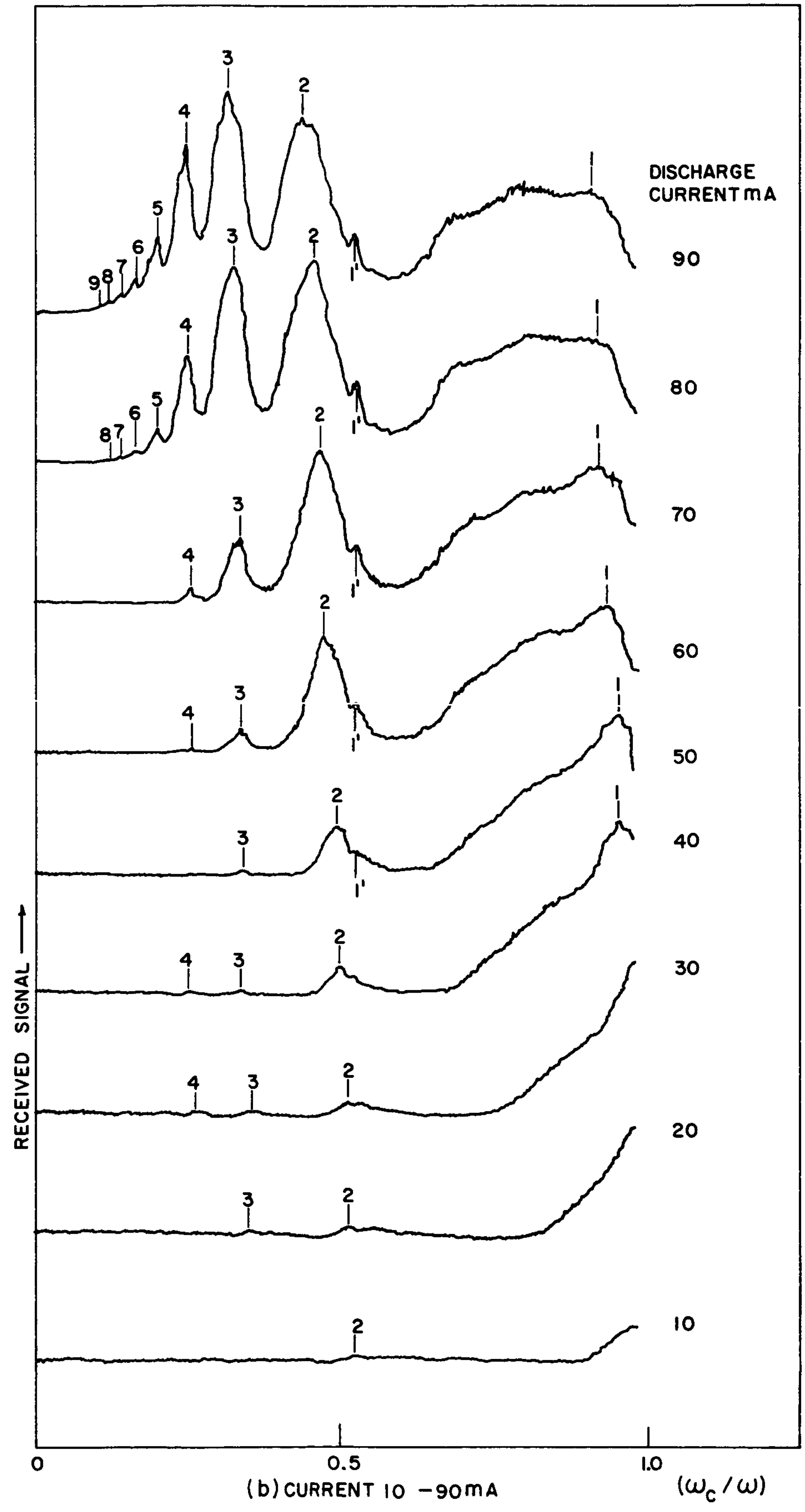

DISCHARGE CURRENT mA
90
80
70
60
50
40
30
20
10
RECEIVED SIGNAL
0
0.5
1.0
(b) CURRENT 10 −90mA
(ω_c/ω)

FIG. 8b

transformation will then give D as a function of $\underset{\sim}{k}$. Now D can be replaced by $(\overleftrightarrow{\epsilon}_p \cdot \underset{\sim}{k}\phi)$ so that ϕ can be determined as a function of $\underset{\sim}{k}$. Transformation of ϕ back to coordinate space gives the values of the potential at the two wires. Since Q and ϕ are now known, the capacitance between the wires is completely determined. Since this quantity is computed using the form of $\overleftrightarrow{\epsilon}_p$ appropriate to a warm magnetoplasma, comparison of the measured and predicted impedance will serve to verify the theory of $\overleftrightarrow{\epsilon}_p$.

Naturally, the analytical steps in the impedance calculation just described are simpler to state than to carry out, and suitable approximations are required. The experiments show qualitatively the features to be expected from such an analysis. To obtain quantitative comparisons, the theoretical aspects must be considered further. This will be done when the theoretical work described in Section II.A has been completed. For the moment, experiments of this nature have been suspended in favor of an alternative approach suggested by Harp.[21,22] This is based on an observation originally made by Tanaka and Kubo,[23] that the usual major peaks in cyclotron harmonic noise radiation are frequently associated with series of small subsidiary peaks lying between them. These were explained by Buchsbaum and Hasegawa[24] as being due to standing cyclotron harmonic waves trapped in the central higher density core of the plasma. This implies that measurements on the peaks can give direct information on the form of the cyclotron harmonic wave dispersion relation. Such measurements were made by Harp[21,22] using a movable probe system to determine wavelength, and varying the parameters ω_c and ω_p . His initial results have shown good agreement between the experimental data and the curves of Fig. 2.

Attempts were made to carry out dispersion measurements of the type described above, but as will be appreciated from Fig. 8 the subsidiary resonances are only very weakly excited in mercury-vapor. For this reason, we have been working recently on the modification of a continuously pumped system in which it is intended to use rare gases. The data of the Japanese group[23] and of Harp[21,22] indicate that under these conditions strong subsidiary resonances are excited. Typical records for the conditions under which our future work will be carried out are shown in Fig. 9.

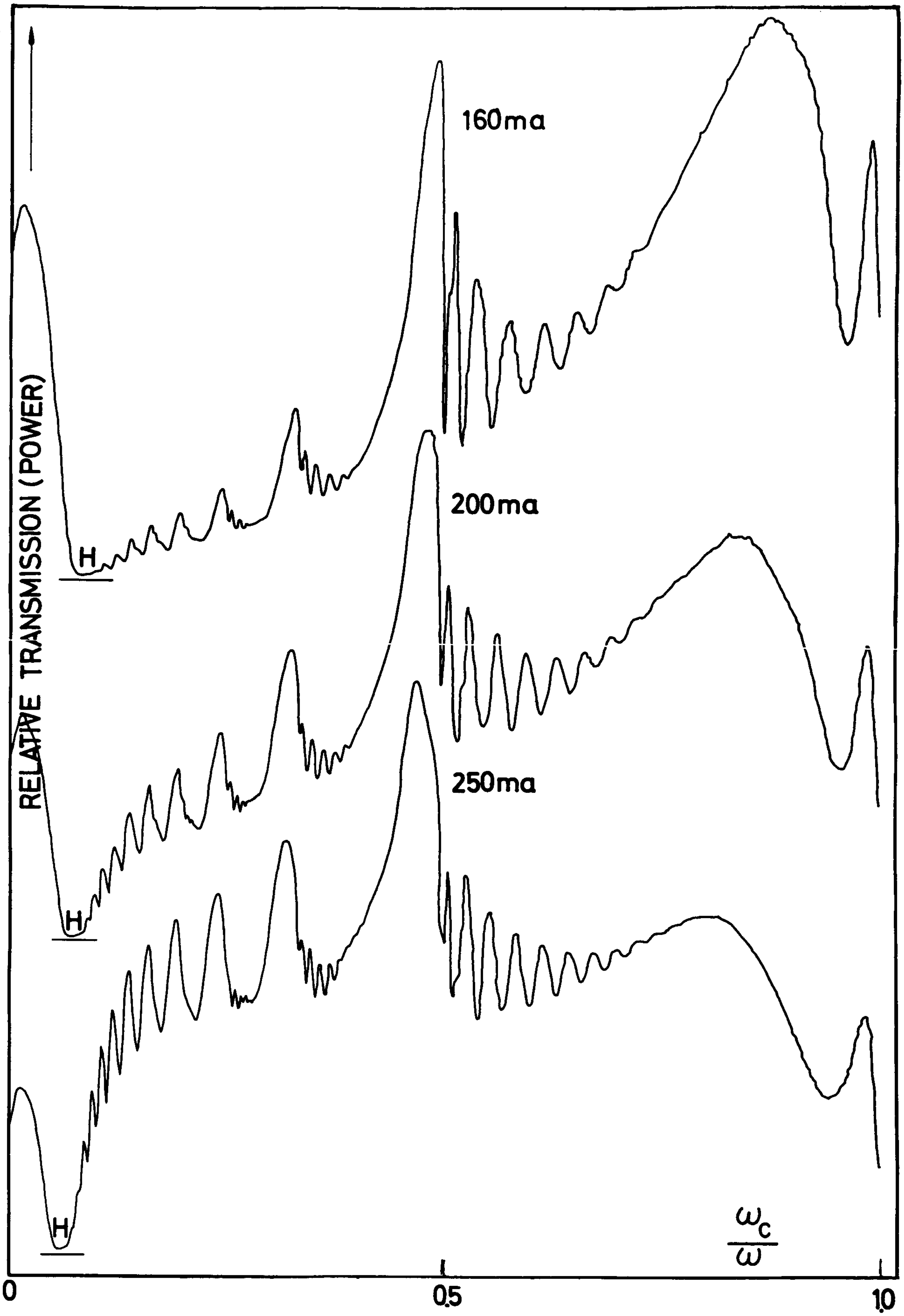
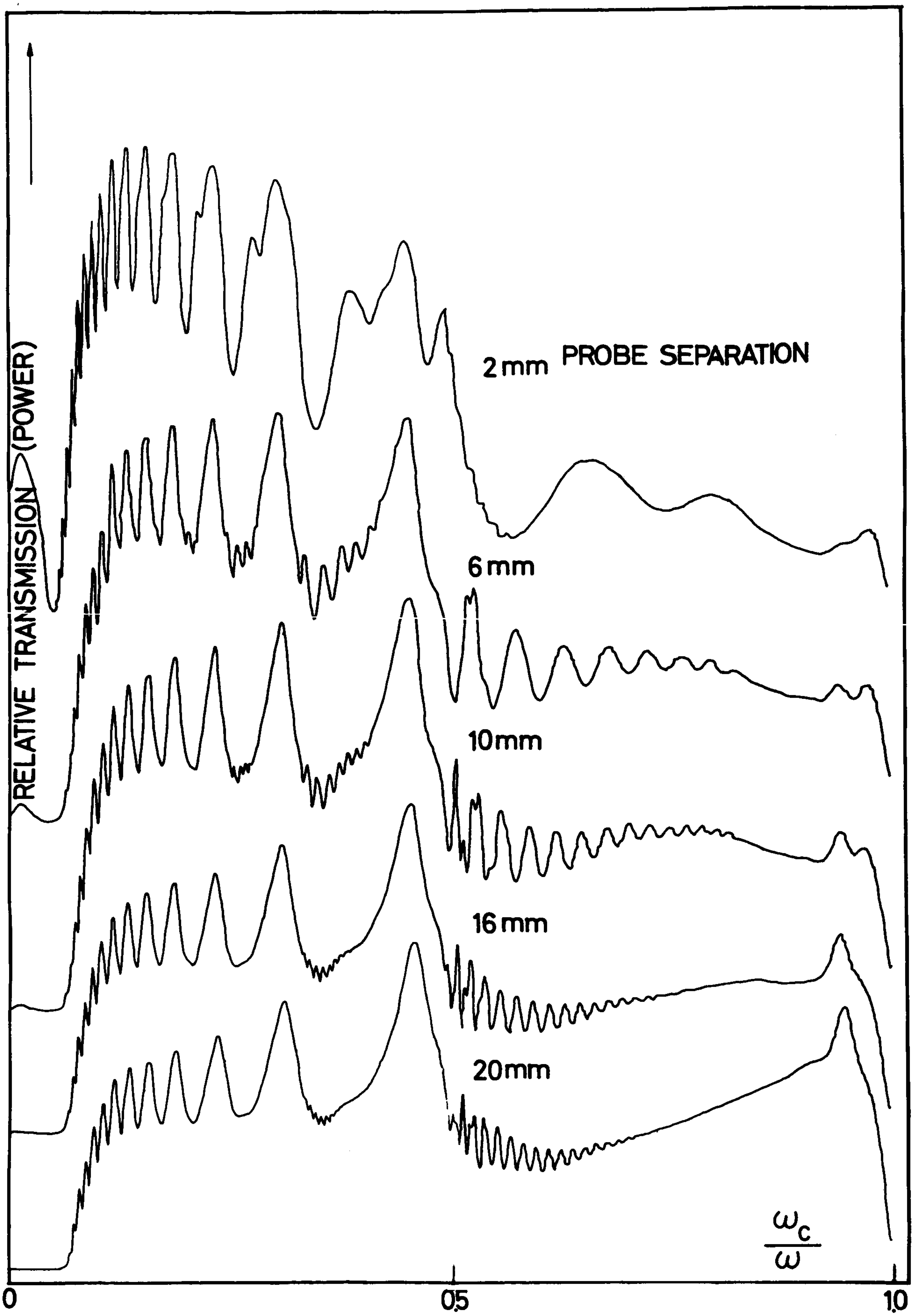


FIG. 9a Transmission curves with varying current and spacing. Argon at 3.10^{-4} mm Hg. (the curves have been separated vertically for charity). (after Harp Ref. 22)



RELATIVE TRANSMISSION (POWER)
2 mm PROBE SEPARATION
6 mm
10 mm
16 mm
20 mm
$\frac{\omega_c}{\omega}$
0
0.5
1.0

FIG. 9b

C. EXPERIMENTS ON "ALOUETTE" PLASMA RESONANCES

Project No. 1309 — F. W. Crawford, R. S. Harp, and H. H. Weiss

The main intention of this project is to reproduce the "Alouette" ringing phenomenon in the laboratory. The principal difficulties encountered in doing this are due to the increased working frequencies required, and the proportionately greater influence of collisions. The satellite transmitter covered the frequency range 0.5 - 12 Mc/s, and was pulsed on for 100 μs. Ringing persisted for a few ms. The local collision frequency would be of the order of 10/sec. It is not feasible to scale these values to the mercury-vapor discharge employed in our initial experiments (see Fig. 7) since the collision frequency is of order 10^7/sec. The maximum frequency that could be used with the modulator available was 450 Mc/s giving $(\nu/\omega) \approx 0.005$. Since a decay time-constant of 100 ns was expected then, due to collisions, and the modulator fall-time was of the order of 100 ns, the experiment was expected to be a marginal one. In the event, no ringing could be observed. An additional source of damping that may have influenced the results was the axial drift of the electrons in the discharge tube. This is of the order of 2.10^7 cm/sec, so that with the 7 cm excitation antenna used "memory" of the pulse would be completely lost in times of the order of 300 ns.

To improve the experimental conditions, a better modulator has been purchased. This will allow us to double the working frequency, and improve the fall-time of the pulse. The experiments are now being prepared in the rare gas set-up described in Section II.B which will also give one to two orders of magnitude lower collision frequency than the mercury-vapor discharge.

An additional series of experiments employing a pulsed microwave signal was attempted in the mercury-vapor discharge tube shown in Fig. 10. The discharge was produced by rf at about 30 Mc., and was immersed in the magnetic field of two large Helmholtz coils. The tube contained three parallel wire dipole antennas. The object of the experiment was to pulse the microwave signal applied to one of these, and to measure

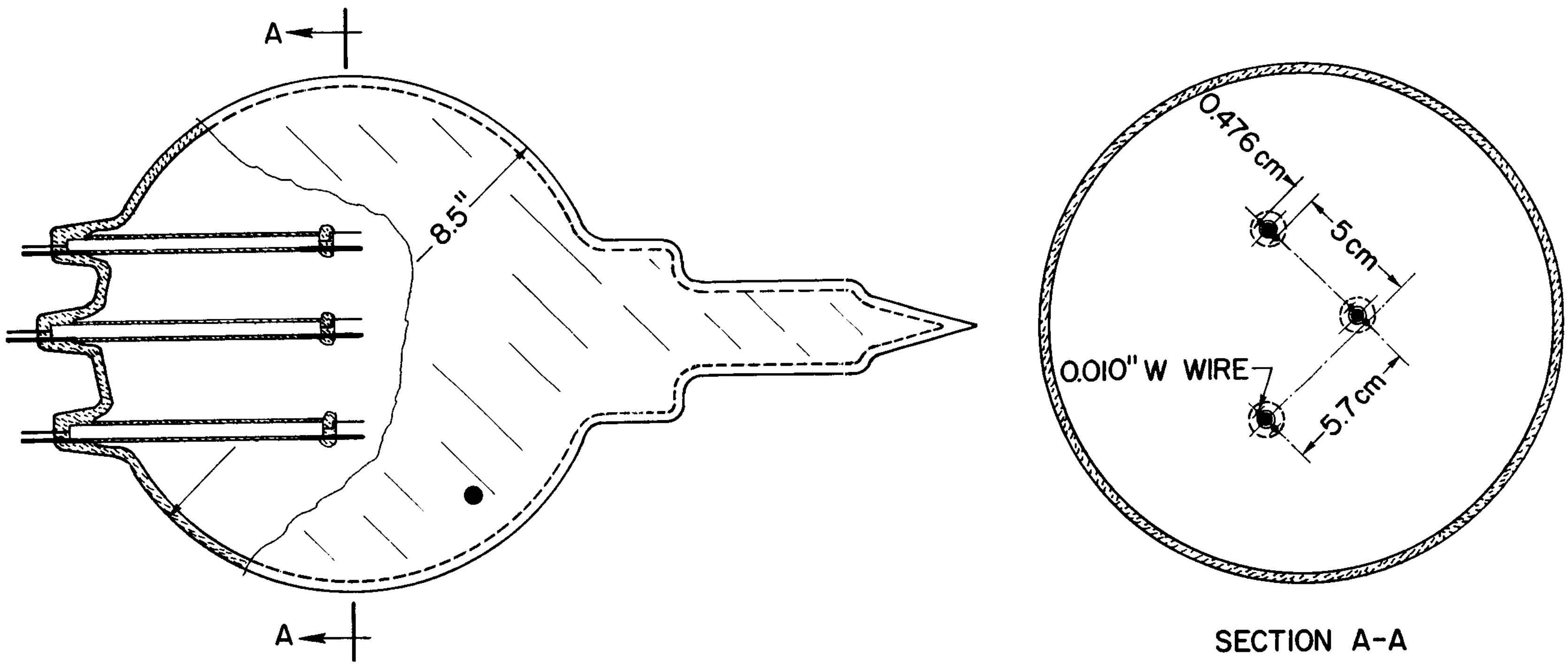


FIG. 10 Experimental discharge tube for transmission measurements on pulsed cyclotron harmonic wave signals

the delay before arrival of the signal at one of the others. This gives a measure of the group velocity, which can then be compared with the slope of the dispersion curves plotted in Fig. 2. Similar difficulties were encountered in this experiment to those described above on the resonances. The attenuation per unit distance is of order (ν/v_g), where the maximum value of the group velocity v_g is only a fraction of the thermal velocity ($\sim 10^8$ cm/s in these experiments). Hence heavy attenuation of the pulse occurred. Difficulties were also encountered in the phasing of the microwave tone-burst so that quantitative measurements were not possible. The main features of Fig. 5 were observed, however, i.e., that the attenuation and group delay were minimum at frequencies slightly in excess of $n\omega_c$. The experimental troubles will be considerably eased in the new set-up, and good qualitative results should be obtainable.

III. FUTURE PROGRAM

In the remaining six months of this research grant, it is hoped to complete the computations of cyclotron and Landau damping for both (complex ω , real $k_\perp$) and (complex $k_\perp$ and real ω) . If there is time, a start will then be made on computing the impedance between two wires in a warm magnetoplasma so that the transmission measurements in Section II.B can be interpreted quantitatively.

On the experimental side, it is expected that the continuously-pumped system being modified for temporary use in this work will be in operation soon, and that the use of argon as the working gas, and signal frequencies of the order of 800 Mc/s, will permit us to carry out precise experiments of the following kinds: first, direct verification of the dispersion curves for perpendicular propagation shown in Fig. 2, second pulsed measurements of ringing, and group velocity, and third, if time permits, an investigation of the resonance probe"[25] potentialities of the cyclotron harmonic resonances.

IV. REPORTS, CONFERENCE PAPERS, AND PUBLICATIONS RESULTING FROM RESEARCH GRANT NGR 05-020-077

1. Tataronis, J. A., and Crawford, F. W., "Cyclotron and Collision Damping of Propagating Waves in a Magnetoplasma,"

   IPR 27 (August 1965).
   *Proc. 7th International Conference on Phenomena in Ionized Gases, Belgrade, Yugoslavia, August 1965 (to be published).

2. Crawford, F. W., "European Travel Report,"
   IPR 35 (October 1965).

3. Semiannual Report No. 1 (1 May - 31 October, 1965)
   IPR 39 (November 1965).

IPR = Institute for Plasma Research Report

* = Conference Presentation.

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